Spiral plat avec courbes terminales externe et interne

Poids du spiral et anisochronisme en position verticale

Cas d'une montre bracelet

Caractéristiques du spiral ***dextre***

Référence :C:\Résonateur (TA)\Data\Bal_spiral plat (ex num).mcd(R)

Référence :C:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $\quad ép = 0.03\,mm \quad ha = 0.15\,mm \quad S = 4.5\times 10^{-3}\,mm^2 \quad TOL := 10^{-12}$

$d2_{sp} = 4.52\,mm \quad d_V := 1.1\cdot mm \quad d_B := 1.312\cdot d1_{sp} \quad p_{sp} = 0.135\,mm \quad n_{sp} := \dfrac{d2_{sp} - d_B}{2\cdot p_{sp}}$

$L := \pi\cdot\dfrac{n_{sp}}{2}\cdot\left(d2_{sp} + d_B\right) \quad L = 10.674\,cm \quad \psi_0 := 2\cdot\pi\cdot n_{sp} \quad \psi_0 = 4.102\times 10^3\,deg$

Position du point de raccordement sur le spiral $\quad \alpha_A := \pi \quad r_A := 0.5\cdot d2_{sp} \quad z_A := r_A\cdot e^{i\cdot\alpha_A}$

Forme initiale du spiral

$a := \dfrac{p_{sp}}{2\cdot\pi} \quad r_s(\alpha) := r_A - a\cdot\left(\alpha - \alpha_A\right) \quad x_{0s}(\alpha) := r_s(\alpha)\cdot cos(\alpha) \quad y_{0s}(\alpha) := r_s(\alpha)\cdot sin(\alpha)$

$s(\alpha) := \dfrac{1}{2\cdot a}\cdot\left(r_A{}^2 - r_s(\alpha)^2\right) \quad s(\alpha) := r_A\cdot\left(\alpha - \alpha_A\right) - \dfrac{a}{2}\cdot\left(\alpha - \alpha_A\right)^2$

Courbe terminale externe

$r_{t1} := 0.8 \quad r_{t1} := racine\left[\left(2\cdot r_{t1} - 1\right)^4 - 4\cdot\left(1 - r_{t1}\right)^4 - \pi^2\cdot r_{t1}{}^2\cdot\left(1 - r_{t1}\right)^2, r_{t1}\right]\cdot r_A \quad r_{t1} = 0.832\,r_A$

$r_{t2} := 2\cdot r_{t1} - r_A \quad r_{t2} = 0.665\,r_A \quad \beta_0 := arctan\left[\dfrac{\pi\cdot r_{t1}}{2\cdot\left(r_A - r_{t1}\right)}\right] \quad \beta_0 = 82.695\,deg \quad l_t := r_{t2}\cdot\beta_0 + \pi\cdot r_{t1}$

$x_{0t1}\left(\alpha_t\right) := -r_A + r_{t1}\cdot\left(1 + cos\left(\alpha_t\right)\right) \quad y_{0t1}\left(\alpha_t\right) := r_{t1}\cdot sin\left(\alpha_t\right)$

$x_{0t2}\left(\beta_t\right) := r_{t2}\cdot cos\left(\beta_t\right) \quad y_{0t2}\left(\beta_t\right) := r_{t2}\cdot sin\left(\beta_t\right)$

Courbe terminale interne $\quad \alpha_B := mod\left(\psi_0 + \pi, 2\cdot\pi\right) \quad \alpha_B = 322.4\,deg \quad r_B := 0.5\cdot d_B$

$\beta := 121\cdot deg \quad \beta'_0 := racine\left[\beta\cdot\left(\sqrt{2}\cdot sin(\beta) - 1\right) + sin(\beta)\cdot cos(\beta), \beta\right] \quad \beta'_0 = 121.21\,deg$

$r_{t'} := \dfrac{r_B}{\sqrt{2}\cdot sin\left(\beta'_0\right)} \quad r_{t'} = 0.827\,r_B \quad x_{0t}\left(\alpha_t\right) := -r_B + r_{t'}\cdot\left(1 + cos\left(\alpha_t\right)\right) \quad y_{0t}\left(\alpha_t\right) := r_{t'}\cdot sin\left(\alpha_t\right)$

$x_{0t'}\left(\alpha_{t'}\right) := \left[r_B + r_{t'}\cdot\left(-1 + cos\left(\alpha_{t'}\right)\right)\right]cos\left(\alpha_B\right) - r_{t'}\cdot sin\left(\alpha_{t'}\right)\cdot sin\left(\alpha_B\right) \quad l_{t'} := r_{t'}\cdot 2\cdot\beta'_0$

$y_{0t'}\left(\alpha_{t'}\right) := \left[r_B + r_{t'}\cdot\left(-1 + cos\left(\alpha_{t'}\right)\right)\right]sin\left(\alpha_B\right) + r_{t'}\cdot sin\left(\alpha_{t'}\right)\cdot cos\left(\alpha_B\right) \quad L_t := l_t + L + l_{t'}$

Position des goupilles de raquettes $\quad r_{GR} := r_{t2} \quad \alpha_{GR} := -\beta_0 \quad \alpha_{GR} = -82.695\,deg$

$x_{GR} := x_{0t2}\left(\alpha_{GR}\right) \quad y_{GR} := y_{0t2}\left(\alpha_{GR}\right)$

Position du point d'attache à la virole $\quad r_V := \sqrt{x_{0t'}\left(2\cdot\beta'_0\right)^2 + y_{0t'}\left(2\cdot\beta'_0\right)^2} \quad \alpha_V(\theta) := Atan\left(x_{0t'}\left(2\cdot\beta'_0\right), y_{0t'}\left(2\cdot\beta'_0\right)\right) + \theta$

$r_V = 0.55\,mm \quad \alpha_V(0) = 216.447\,deg \quad x_V(\theta) := r_V\cdot cos\left(\alpha_V(\theta)\right) \quad y_V(\theta) := r_V\cdot sin\left(\alpha_V(\theta)\right)$

Amplitude stationnaire du balancier $\quad \theta_0 = 270\,deg$

Moment quadratique de section

Référence :C:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha)$

Calcul du déplacement de centre de gravité

$z_{0s}(\alpha) := x_{0s}(\alpha) + i \cdot y_{0s}(\alpha)$ $\quad z_{0t1}(\alpha_t) := x_{0t1}(\alpha_t) + i \cdot y_{0t1}(\alpha_t)$ $\quad z_{0t2}(\beta_t) := x_{0t2}(\beta_t) + i \cdot y_{0t2}(\beta_t)$

$$\sigma 2 := \frac{1}{L_t} \cdot \left[\int_{\pi}^{\pi+\psi_0} \left(\left| z_{0s}(\alpha) \right| \right)^2 \cdot r_s(\alpha)\, d\alpha + \int_0^{\pi} \left(\left| z_{0t1}(\alpha_t) \right| \right)^2 \cdot r_{t1}\, d\alpha_t + \int_{-\beta_0}^{0} \left(\left| z_{0t2}(\beta_t) \right| \right)^2 \cdot r_{t2}\, d\beta_t \right]$$

$z_{0t'}(\alpha_t) := x_{0t'}(\alpha_t) + i \cdot y_{0t'}(\alpha_t)$ $\quad \sigma 2 := \sigma 2 + \frac{1}{L_t} \cdot \int_0^{2 \cdot \beta'_0} \left(\left| z_{0t'}(\alpha_{t'}) \right| \right)^2 \cdot r_{t'}\, d\alpha_{t'}$ $\quad \sigma 2 = 2.794\, mm^2$

$s_s(\alpha) := s(\alpha) + l_t$ $\quad \kappa_s := \frac{1}{\sigma 2 \cdot L_t^2} \cdot \int_{\pi}^{\pi+\psi_0} s_s(\alpha) \cdot \left(\left| z_{0s}(\alpha) \right| \right)^2 \cdot r_s(\alpha)\, d\alpha$

$s_{t2}(\beta_t) := r_{t2} \cdot (\beta_0 + \beta_t)$ $\quad s_{t1}(\alpha_t) := (r_{t2} \cdot \beta_0 + r_{t1} \cdot \alpha_t)$

$$\kappa_t := \frac{1}{\sigma 2 \cdot L_t^2} \cdot \left[\int_{-\beta_0}^{0} s_{t2}(\beta_t) \cdot \left(\left| z_{0t2}(\beta_t) \right| \right)^2 \cdot r_{t2}\, d\beta_t + \int_0^{\pi} s_{t1}(\alpha_t) \cdot \left(\left| z_{0t1}(\alpha_t) \right| \right)^2 \cdot r_{t1}\, d\alpha_t \right]$$

$s_{t'}(\alpha_{t'}) := r_{t'} \cdot \alpha_{t'} + L + l_t$

$\kappa_{t'} := \frac{1}{\sigma 2 \cdot L_t^2} \cdot \left[\int_0^{2 \cdot \beta'_0} s_{t'}(\alpha_{t'}) \cdot \left(\left| z_{0t'}(\alpha_{t'}) \right| \right)^2 \cdot r_{t'}\, d\alpha_{t'} \right]$ $\quad \kappa := \kappa_t + \kappa_s + \kappa_{t'}$ $\quad \kappa = 0.372$

$\mathbf{\Delta_s}(\theta) := \frac{i \cdot \theta}{L_t} \cdot \int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha) \cdot exp\left(i \cdot \theta \cdot \frac{s_s(\alpha)}{L_t} \right) \cdot r_s(\alpha)\, d\alpha$ $\quad \mathbf{\Delta_s}(\theta_0) = 0.164 + 0.043i\, mm$

$$\mathbf{\Delta_t}(\theta) := \frac{i \cdot \theta}{L_t} \cdot \left(\int_{-\beta_0}^{0} z_{0t2}(\beta_t) \cdot exp\left(i \cdot \frac{\theta}{L_t} \cdot s_{t2}(\beta_t) \right) \cdot r_{t2}\, d\beta_t + \int_0^{\pi} z_{0t1}(\alpha_t) \cdot exp\left(i \cdot \frac{\theta}{L_t} \cdot s_{t1}(\alpha_t) \right) \cdot r_{t1}\, d\alpha_t \right)$$

$\mathbf{\Delta_t}(\theta_0) = -0.164 - 0.049i\, mm$

$\mathbf{\Delta_{t'}}(\theta) := \frac{i \cdot \theta}{L_t} \cdot \left(\int_0^{2 \cdot \beta'_0} z_{0t'}(\alpha_{t'}) \cdot exp\left(i \cdot \frac{\theta}{L_t} \cdot s_{t'}(\alpha_{t'}) \right) \cdot r_{t'}\, d\alpha_{t'} \right)$ $\quad \mathbf{\Delta_{t'}}(\theta_0) = 0.013 + 0.015i\, mm$

$\mathbf{\Delta_1}(\theta) := \mathbf{\Delta_t}(\theta) + \mathbf{\Delta_s}(\theta) + \mathbf{\Delta_{t'}}(\theta)$ $\quad \mathbf{\Delta_1}(\theta_0) = 0.014 + 8.409i \times 10^{-3}\, mm$

$\zeta(\theta) := -i \cdot \frac{d}{d\theta} \mathbf{\Delta_1}(\theta) - \kappa \cdot \mathbf{\Delta_1}(\theta)$ $\quad \zeta(\theta_0) = -2.206 \times 10^{-3} - 7.88i \times 10^{-3}\, mm$

$$\zeta_s(\theta) := \frac{1}{L_t}\cdot\int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha)\cdot e^{i\cdot\theta\cdot\frac{s_s(\alpha)}{L_t}}\cdot\left[1 + i\cdot\theta\cdot\left(\frac{s_s(\alpha)}{L_t} - \kappa\right)\right]\cdot r_s(\alpha)\, d\alpha$$

$$\zeta_{t2}(\theta) := \frac{1}{L_t}\cdot\int_{-\beta_0}^{0} z_{0t2}(\beta_t)\cdot e^{i\cdot\theta\cdot\frac{s_{t2}(\beta_t)}{L_t}}\cdot\left[1 + i\cdot\theta\cdot\left(\frac{s_{t2}(\beta_t)}{L_t} - \kappa\right)\right]\cdot r_{t2}\, d\beta_t$$

$$\zeta_{t1}(\theta) := \frac{1}{L_t}\cdot\int_{0}^{\pi} z_{0t1}(\alpha_t)\cdot e^{i\cdot\theta\cdot\frac{s_{t1}(\alpha_t)}{L_t}}\cdot\left[1 + i\cdot\theta\cdot\left(\frac{s_{t1}(\alpha_t)}{L_t} - \kappa\right)\right]\cdot r_{t1}\, d\alpha_t \qquad \zeta_t(\theta) := \zeta_{t2}(\theta) + \zeta_{t1}(\theta)$$

$$\zeta_{t'}(\theta) := \frac{1}{L_t}\cdot\int_{0}^{2\cdot\beta'_0} z_{0t'}(\alpha_{t'})\cdot e^{i\cdot\theta\cdot\frac{s_{t'}(\alpha_{t'})}{L_t}}\cdot\left[1 + i\cdot\theta\cdot\left(\frac{s_{t'}(\alpha_{t'})}{L_t} - \kappa\right)\right]\cdot r_{t'}\, d\alpha_{t'}$$

$$\zeta(\theta) := \zeta_t(\theta) + \zeta_s(\theta) + \zeta_{t'}(\theta) \qquad \zeta(\theta_0) = -2.206\times10^{-3} - 7.88i\times10^{-3}\,mm$$

Approximations de Haag

Paramètres de la courbe terminale externe

$$X_{0t1}(\alpha_t) := r_A - r_{t1} + r_{t1}\cdot cos(\alpha_t) \qquad Y_{0t1}(\alpha_t) := r_{t1}\cdot sin(\alpha_t) \qquad X_{0t2}(\beta_t) := -r_{t2}\cdot cos(\beta_t) \qquad Y_{0t2}(\beta_t) := -r_{t2}\cdot sin(\beta_t)$$

$$X_1 := \frac{1}{r_A{}^2}\cdot\left(\int_0^{\pi} X_{0t1}(\alpha)\cdot r_{t1}\, d\alpha + \int_0^{\beta_0} X_{0t2}(\beta)\cdot r_{t2}\, d\beta\right) \qquad X_1 = 0$$

$$Y_1 := \frac{1}{r_A{}^2}\cdot\left(\int_0^{\pi} Y_{0t1}(\alpha)\cdot r_{t1}\, d\alpha + \int_0^{\beta_0} Y_{0t2}(\beta)\cdot r_{t2}\, d\beta\right) - 1 \qquad Y_1 = 0$$

$$\rho_1 := \sqrt{X_1{}^2 + Y_1{}^2} \qquad \varphi_1 := Atan(X_1, Y_1) \qquad \rho_1 = 0 \qquad \varphi_1 = 270\,deg$$

$$X_2 := \frac{1}{r_A{}^3}\cdot\left[\int_0^{\pi} r_{t1}\cdot\alpha\cdot X_{0t1}(\alpha)\cdot r_{t1}\, d\alpha + \int_0^{\beta_0} (r_{t1}\cdot\pi + r_{t2}\cdot\beta)\cdot X_{0t2}(\beta)\cdot r_{t2}\, d\beta\right] + 1$$

$$Y_2 := \frac{1}{r_A{}^3}\cdot\left[\int_0^{\pi} r_{t1}\cdot\alpha\cdot Y_{0t1}(\alpha)\cdot r_{t1}\, d\alpha + \int_0^{\beta_0} (r_{t1}\cdot\pi + r_{t2}\cdot\beta)\cdot Y_{0t2}(\beta)\cdot r_{t2}\, d\beta\right]$$

$$\rho_2 := \sqrt{X_2{}^2 + Y_2{}^2} \qquad \varphi_2 := Atan(X_2, Y_2) \qquad \rho_2 = 1.055 \qquad \varphi_2 = 147.579\,deg$$

Paramètres de la courbe terminale interne

$X_{0t'}(\alpha_{t'}) := r_B - r_{t'} + r_{t'} \cdot cos(\alpha_{t'})$ $\quad Y_{0t'}(\alpha_{t'}) := r_{t'} \cdot sin(\alpha_{t'})$ $\quad Z_{0t'}(\alpha) := X_{0t'}(\alpha) + i \cdot Y_{0t'}(\alpha)$

$$Z'_1 := \frac{1}{r_B^{\,2}} \cdot \int_0^{2 \cdot \beta'_0} Z_{0t'}(\alpha) \cdot r_{t'}\, d\alpha - i \qquad \rho'_1 := |Z'_1| \qquad \varphi'_1 := arg(Z'_1)$$

$\rho'_1 = 0$ $\qquad \varphi'_1 = -25.264\, deg$

$$Z'_2 := \frac{1}{r_B^{\,3}} \cdot \int_0^{2 \cdot \beta'_0} r_{t'} \cdot \alpha \cdot Z_{0t'}(\alpha) \cdot r_{t'}\, d\alpha + 1 \qquad \rho'_2 := |Z'_2| \qquad \varphi'_2 := arg(Z'_2)$$

$\rho'_2 = 1.074$ $\qquad \varphi'_2 = 145.651\, deg$

Formule de Haag avec κ = 1/3 $\qquad \boldsymbol{OA} := r_A \cdot e^{i \cdot \pi}$ $\qquad \boldsymbol{OB} := r_B \cdot e^{i \cdot (\pi + \psi_0)}$

$$\zeta_{1ah}(\theta) := \frac{1}{3 \cdot L_t} \cdot \left[r_A \cdot \rho_1 \cdot e^{-i \cdot \varphi_1} \cdot (3 - i \cdot \theta) - \frac{\theta}{L_t} \cdot r_A^{\,2} \cdot \rho_2 \cdot e^{-i \cdot \varphi_2} \cdot (6 \cdot i + \theta) \right] \cdot \boldsymbol{OA}$$

$$\zeta_{2ah}(\theta) := \frac{1}{3 \cdot L_t} \cdot \left[r_B \cdot \rho'_1 \cdot e^{i \cdot \varphi'_1} \cdot (3 + 2 \cdot i \cdot \theta) + \frac{\theta}{L_t} \cdot r_B^{\,2} \cdot \rho'_2 \cdot e^{i \cdot \varphi'_2} \cdot (6 \cdot i - 2 \cdot \theta) \right] \cdot \boldsymbol{OB} \cdot e^{i \cdot \theta} \qquad \zeta_{ah}(\theta) := \zeta_{1ah}(\theta) + \zeta_{2ah}(\theta)$$

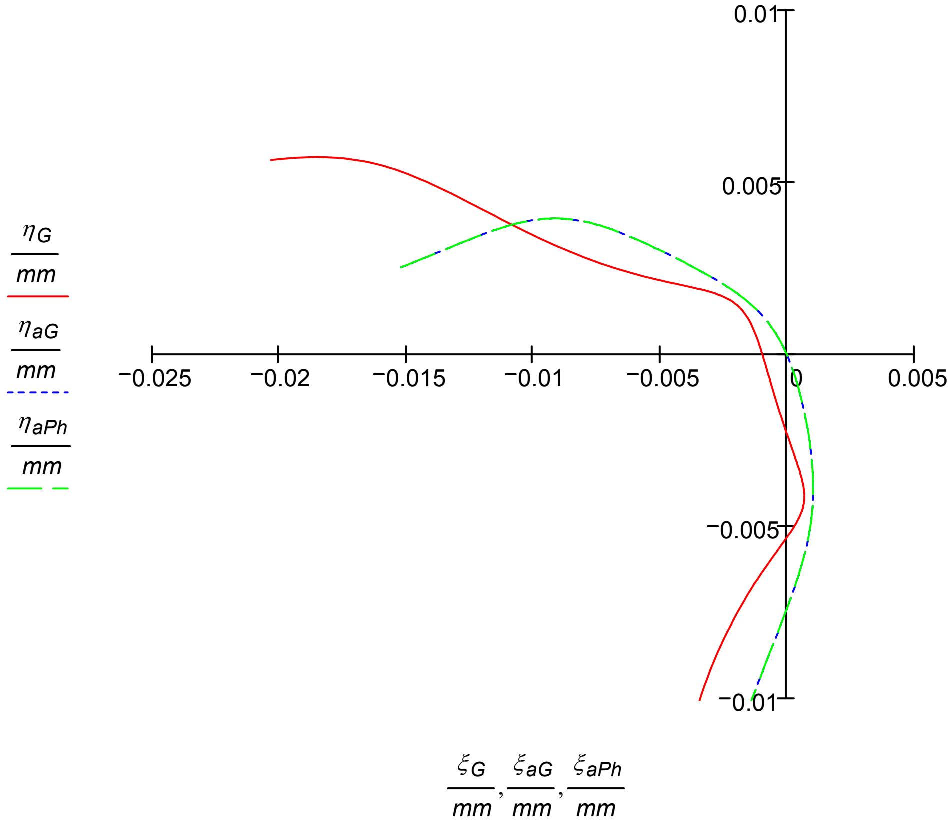
Courbes Phillips

$$\zeta_{aPh}(\theta) := \frac{\theta}{3 \cdot L_t^{\,2}} \cdot \left[-r_A^{\,2} \cdot \rho_2 \cdot e^{-i \cdot \varphi_2} \cdot (6 \cdot i + \theta) \cdot \boldsymbol{OA} + r_B^{\,2} \cdot \rho'_2 \cdot e^{i \cdot \varphi'_2} \cdot (6 \cdot i - 2 \cdot \theta) \cdot \boldsymbol{OB} \cdot e^{i \cdot \theta} \right]$$

Graphes du déplacement du centre de gravité

$n := 201$ $\quad i := 0 .. n - 1$ $\quad \Delta\theta := \frac{4 \cdot \pi}{n - 1}$ $\quad \theta_i := -2 \cdot \pi + i \cdot \Delta\theta$ $\quad \xi_{G_i} := Re(\zeta(\theta_i))$ $\quad \eta_{G_i} := Im(\zeta(\theta_i))$

$\xi_{aG_i} := Re(\zeta_{ah}(\theta_i))$ $\quad \eta_{aG_i} := Im(\zeta_{ah}(\theta_i))$ $\quad \xi_{aPh_i} := Re(\zeta_{aPh}(\theta_i))$ $\quad \eta_{aPh_i} := Im(\zeta_{aPh}(\theta_i))$



Perturbation de période - spiral non déformé en position de repos

Calcul par intégrations numériques

$$\eta(\theta) := Im(\zeta(\theta)) \qquad Gamma(\theta) := -m_s \cdot g \cdot \frac{d}{d\theta}\eta(\theta)$$

$$\theta(\varphi) := \theta_0 \cdot cos(\varphi) \qquad Delta(\theta_0) := \frac{L}{2 \cdot \pi \cdot \theta_0 \cdot E \cdot I_{33}} \cdot \int_0^{2 \cdot \pi} Gamma(\theta_0 \cdot cos(\varphi)) \cdot cos(\varphi \qquad Delta(\theta_0) = 3.661 \times 10^{-6}$$

$$Z_s(\theta_0) := \frac{1}{L_t^2} \cdot \int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha) \cdot s_s(\alpha) \cdot \left[\left(\kappa - \frac{s_s(\alpha)}{L_t}\right) \cdot J0\left(\theta_0 \cdot \frac{s_s(\alpha)}{L_t}\right) - \frac{1}{\theta_0} \cdot J1\left(\theta_0 \cdot \frac{s_s(\alpha)}{L_t}\right)\right] \cdot r_s(\alpha)\, d\alpha$$

$$Z_{t2}(\theta_0) := \frac{r_{t2}}{L_t^2} \cdot \int_{-\beta_0}^{0} z_{0t2}(\beta_t) \cdot s_{t2}(\beta_t) \cdot \left[\left(\kappa - \frac{s_{t2}(\beta_t)}{L_t}\right) \cdot J0\left(\theta_0 \cdot \frac{s_{t2}(\beta_t)}{L_t}\right) - \frac{1}{\theta_0} \cdot J1\left(\theta_0 \cdot \frac{s_{t2}(\beta_t)}{L_t}\right)\right] d\beta_t$$

$$Z_{t1}(\theta_0) := \frac{r_{t1}}{L_t^2} \cdot \int_{0}^{\pi} z_{0t1}(\alpha_t) \cdot s_{t1}(\alpha_t) \cdot \left[\left(\kappa - \frac{s_{t1}(\alpha_t)}{L_t}\right) \cdot J0\left(\theta_0 \cdot \frac{s_{t1}(\alpha_t)}{L_t}\right) - \frac{1}{\theta_0} \cdot J1\left(\theta_0 \cdot \frac{s_{t1}(\alpha_t)}{L_t}\right)\right] d\alpha_t$$

$$Z_{t'}(\theta_0) := \frac{r_{t'}}{L_t^2} \cdot \int_{0}^{2 \cdot \beta'_0} z_{0t'}(\alpha_{t'}) \cdot s_{t'}(\alpha_{t'}) \cdot \left[\left(\kappa - \frac{s_{t'}(\alpha_{t'})}{L_t}\right) \cdot J0\left(\theta_0 \cdot \frac{s_{t'}(\alpha_{t'})}{L_t}\right) - \frac{1}{\theta_0} \cdot J1\left(\theta_0 \cdot \frac{s_{t'}(\alpha_{t'})}{L_t}\right)\right] d\alpha_{t'}$$

$$Z(\theta_0) := Z_{t2}(\theta_0) + Z_{t1}(\theta_0) + Z_s(\theta_0) + Z_{t'}(\theta_0) \qquad Z(\theta_0) = -3.366 \times 10^{-4} - 6.254i \times 10^{-5}\, mm$$

$$Delta(\theta_0) := -g \cdot \frac{m_s \cdot L}{E \cdot I_{33}} \cdot Im(Z(\theta_0)) \qquad Delta(\theta_0) = 3.661 \times 10^{-6}$$

$$\mu(\theta_0) := -86400 \cdot Delta(\theta_0) \qquad \boxed{\mu(\theta_0) = -0.316} \qquad \boxed{\mu(180 \cdot deg) = -0.284}$$

Approximations de Haag

$$f(\theta_0, s) := \frac{s}{L_t} \cdot \left[\left(\kappa - \frac{s}{L_t}\right) \cdot J0\left(\theta_0 \cdot \frac{s}{L_t}\right) - \frac{1}{\theta_0} \cdot J1\left(\theta_0 \cdot \frac{s}{L_t}\right)\right] \qquad f_1(\theta_0, s) := \frac{d}{ds} f(\theta_0, s)$$

$$Z_a(\theta_0) := \frac{1}{L_t} \cdot \left[\left(r_A \cdot \rho_1 \cdot e^{-i \cdot \varphi_1} + 2 \cdot a\right) \cdot f(\theta_0, l_t) - r_A^2 \cdot \rho_2 \cdot e^{-i \cdot \varphi_2} \cdot f_1(\theta_0, l_t)\right] \cdot \mathbf{OA}$$

$$Z_a(\theta_0) := Z_a(\theta_0) + \frac{1}{L_t} \cdot \left[\left(r_B \cdot \rho'_1 \cdot e^{i \cdot \varphi'_1} - 2 \cdot a\right) \cdot f(\theta_0, l_t + L) + r_B^2 \cdot \rho'_2 \cdot e^{i \cdot \varphi'_2} \cdot f_1(\theta_0, l_t + L)\right] \cdot \mathbf{OB}$$

$$\delta_a(\theta_0) := -g \cdot \frac{m_s \cdot L}{E \cdot I_{33}} \cdot Im(Z_a(\theta_0)) \qquad \delta_a(\theta_0) = 1.853 \times 10^{-6}$$

$$\mu_a(\theta_0) := -86400 \cdot \delta_a(\theta_0) \qquad \boxed{\mu_a(\theta_0) = -0.16} \qquad \boxed{\mu_a(180 \cdot deg) = -0.134}$$

$$F(\theta_0) := J0(\theta_0) - \theta_0 \cdot J1(\theta_0) \qquad F_1(\theta_0) := (1 - \kappa) \cdot J0(\theta_0) + \frac{1}{\theta_0} \cdot J1(\theta_0) \qquad F_2(\theta_0) := 2 \cdot J0(\theta_0) + (1 - \kappa) \cdot F(\theta_0)$$

$$Z_{ah}(\theta_0) := \frac{\kappa}{L_t^{2}}\cdot\left[\left(r_A\cdot\rho_1\cdot e^{-i\cdot\varphi_1}+2\cdot a\right)\cdot l_t-r_A^{2}\cdot\rho_2\cdot e^{-i\cdot\varphi_2}\right]\cdot \mathbf{OA}$$

$$Z_{ah}(\theta_0) := Z_{ah}(\theta_0)-\frac{1}{L_t}\cdot\left[\left(r_B\cdot\rho'_1\cdot e^{i\cdot\varphi'_1}-2\cdot a\right)\cdot F_1(\theta_0)+\frac{r_B^{2}}{L_t}\cdot\rho'_2\cdot e^{i\cdot\varphi'_2}\cdot F_2(\theta_0)\right]\cdot \mathbf{OB}$$

$$\delta_{ah}(\theta_0) := -g\cdot\frac{m_s\cdot L}{E\cdot I_{33}}\cdot Im\left(Z_{ah}(\theta_0)\right) \qquad \delta_{ah}(\theta_0) = 8.419\times 10^{-6}$$

$$\mu_{ah}(\theta_0) := -86400\cdot\delta_{ah}(\theta_0) \qquad \boxed{\mu_{ah}(\theta_0) = -0.727} \qquad \boxed{\mu_{ah}(180\cdot deg) = -0.62}$$

Cas de courbes Phillips

$$Z_{Ph}(\theta_0) := \frac{\kappa}{L_t^{2}}\cdot\left(2\cdot a\cdot l_t-r_A^{2}\cdot\rho_2\cdot e^{-i\cdot\varphi_2}\right)\cdot \mathbf{OA}-\frac{1}{L_t}\cdot\left[(-2\cdot a)\cdot F_1(\theta_0)+\frac{r_B^{2}}{L_t}\cdot\rho'_2\cdot e^{i\cdot\varphi'_2}\cdot F_2(\theta_0)\right]\cdot \mathbf{OB}$$

$$\delta_{Ph}(\theta_0) := -g\cdot\frac{m_s\cdot L}{E\cdot I_{33}}\cdot Im\left(Z_{Ph}(\theta_0)\right) \qquad \delta_{Ph}(\theta_0) = 8.419\times 10^{-6}$$

$$\mu_{Ph}(\theta_0) := -86400\cdot\delta_{Ph}(\theta_0) \qquad \boxed{\mu_{Ph}(\theta_0) = -0.727} \qquad \boxed{\mu_{Ph}(180\cdot deg) = -0.62}$$

Approximations supplémentaires avec κ = 1/3 et $l_t \ll L_t$

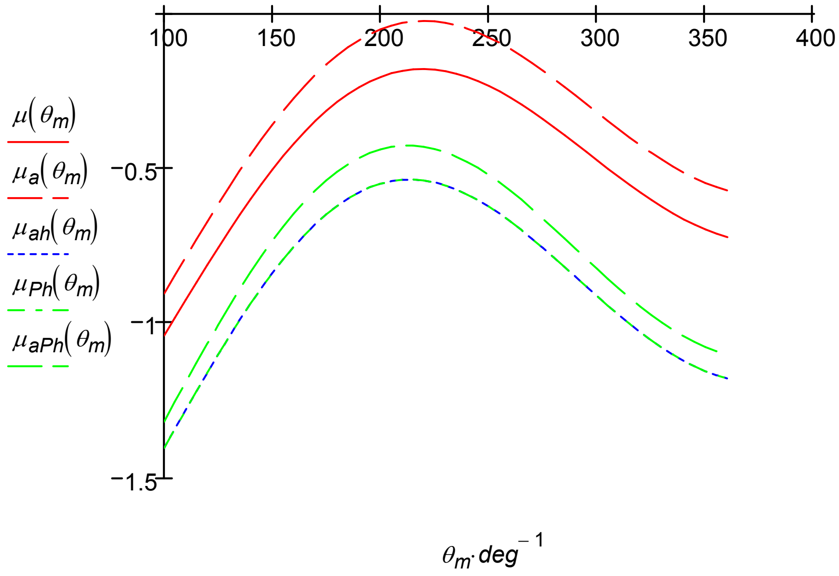
$$U(\theta_0) := \frac{2}{3}\cdot J0(\theta_0)+\frac{1}{\theta_0}\cdot J1(\theta_0)$$

$$Z_{aPh}(\theta_0) := \frac{-r_A^{2}}{3\cdot L_t^{2}}\cdot\rho_2\cdot e^{-i\cdot\varphi_2}\cdot \mathbf{OA}-\frac{2\cdot r_B^{2}}{3\cdot L_t^{2}}\cdot\rho'_2\cdot e^{i\cdot\varphi'_2}\cdot\left(3\cdot J0(\theta_0)+F(\theta_0)\right)\cdot \mathbf{OB}+\frac{2\cdot a}{L_t}\cdot U(\theta_0)\cdot \mathbf{OB}$$

$$\delta_{aPh}(\theta_0) := -g\cdot\frac{m_s\cdot L}{E\cdot I_{33}}\cdot Im\left(Z_{aPh}(\theta_0)\right) \qquad \delta_{aPh}(\theta_0) = 7.304\times 10^{-6}$$

$$\mu_{aPh}(\theta_0) := -86400\cdot\delta_{aPh}(\theta_0) \qquad \boxed{\mu_{aPh}(\theta_0) = -0.631} \qquad \boxed{\mu_{aPh}(180\cdot deg) = -0.51}$$

$$\theta_m := 100\cdot deg, 105\cdot deg .. 360\cdot deg$$

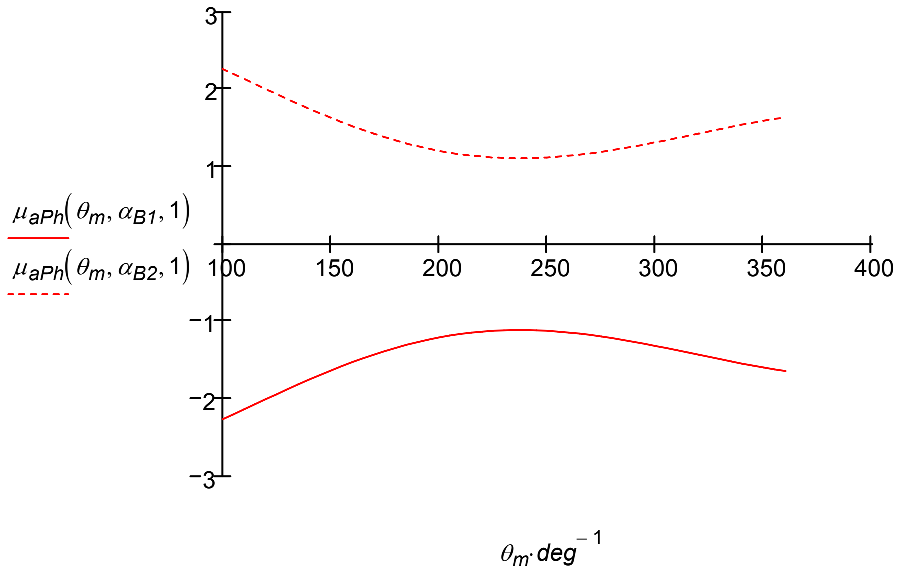


Influence de la position du point de raccordement de la courbe terminale interne

$$\mathbf{OB}(\alpha_B) := r_B \cdot e^{i \cdot \alpha_B} \qquad \alpha_{B1} := \frac{3 \cdot \pi}{2} \qquad \alpha_{B2} := \frac{\pi}{2} \qquad \varphi_2(k, p) := 2 \cdot k \cdot \pi + p \cdot \pi - \psi_0$$

$$Z_{aPh}(\theta_0, \alpha_B, p) := \left(\frac{-r_A^{\,2}}{3 \cdot L_t^{\,2}} \cdot \rho_2 \cdot e^{i \cdot p \cdot \pi} \cdot \frac{r_A}{r_B} + \frac{2 \cdot a}{L_t} \cdot U(\theta_0) \right) \cdot \mathbf{OB}(\alpha_B)$$

$$\delta_{aPh}(\theta_0, \alpha_B, p) := -g \cdot \frac{m_s \cdot L}{E \cdot I_{33}} \cdot Im\left(Z_{aPh}(\theta_0, \alpha_B, p)\right) \qquad \mu_{aPh}(\theta_0, \alpha_B, p) := -86400 \cdot \delta_{aPh}(\theta_0, \alpha_B, p)$$



Spiral plat
avec courbes terminales